Gauging the three-nucleon system [1]

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Introduction

We show how to attach an external photon to a system of three strongly interacting nucleons described by three-body scattering equations. Our method involves the idea of gauging the scattering equations themselves, and results in electromagnetic amplitudes where the external photon is effectively coupled to every part of every strong interaction diagram in the model. Current conservation is therefore implemented in the theoretically correct fashion. In this way we obtain the expressions needed to calculate all possible electromagnetic processes of the three-nucleon system including the electromagnetic form factors of the three-body bound state, $pd \to pd\gamma$, $\gamma^3\mathrm{He} \to pd$, $\gamma^3\mathrm{He} \to ppn$, etc. As the photon is coupled everywhere in the strong interaction model, a unified description of the NNN-γNNN system is obtained. An interesting aspect of our results is the natural appearance of subtraction terms needed to avoid the overcounting of diagrams. We have also applied the gauging of equations method to the four-dimensional description of the πNN system which, because of the possibility of pion absorption, has overcounting problems all of its own, and is therefore much more complicated than the NNN case discussed here. Despite these complications, the gauging procedure itself is very simple and leads easily to a unified description of the πNN-$\gamma\pi NN$ system.[1]

Gauging the three-nucleon Green function

In this presentation we work within the covariant framework of quantum field theory. For three distinguishable nucleons, the Green function G is expressed in terms of its fully disconnected part G_0, and the kernel V, by

$$G = G_0 + G_0 V G. \tag{1}$$

This equation is basically a topological statement regarding the three-particle irreducible structure of Feynman diagrams belonging to G; as such, it can be utilised directly to express the structure of the same Feynman diagrams, but with photons attached everywhere. Thus from Eq. (1) it immediately follows that

$$G^\mu = G_0^\mu + G_0^\mu V G + G_0 V^\mu G + G_0 V G^\mu. \tag{2}$$

This result expresses G^μ in terms of an integral equation, and illustrates what we mean by *gauging an equation*, in this case the gauging of Eq. (1). Note that G^μ consists of all the Feynman diagrams belonging to G where a photon has been attached to all possible places. Neglecting three-body forces, we follow the usual formulation of the three-body problem and write $V = V_1 + V_2 + V_3$ with $V_i = v_i d_i^{-1}$ where d_i is the propagator of nucleon i, and v_i is the two-body potential between particles j and k, $(i \neq j, k)$. By solving Eq. (2) for G^μ and further gauging the above equation for V_i, we obtain that

$$G^\mu = G\Gamma^\mu G \tag{3}$$

where Γ^μ is the three-nucleon electromagnetic vertex function given by

$$\Gamma^\mu = \sum_{i=1}^{3} \left(\Gamma_i^\mu D_{0i}^{-1} + v_i^\mu d_i^{-1} - v_i \Gamma_i^\mu \right). \tag{4}$$

[1]Contributed talk at XVth International Conference on Few-Body Problems in Physics, Groningen, 1997, http://www.kvi.nl/disk$1/fbxv/www/abs_list_num.html

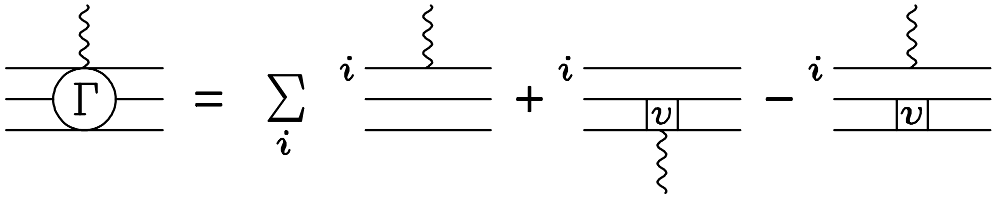

Figure 1: Illustration of Eq. (4) for the three-nucleon electromagnetic vertex function Γ^μ.

Here D_{0i} is the free two-body Green function for particles j and k, Γ^μ_i is the i'th nucleon's electromagnetic vertex function, and v^μ_i is the gauged two-body potential. Eq. (4) is illustrated in Fig. 1. It is interesting to note the natural appearance of the subtraction term $-v_i\Gamma^\mu_i$. When used to calculate G^μ, the presence of this subtraction term in Γ^μ stops the overcounting of diagrams coming from the relativistic impulse approximation term (first term on the r.h.s. of Eq. (4)). By taking appropriate residues of Eq. 3, we can obtain expressions for all possible electromagnetic observables of three nucleons. In the pioneering four-dimensional calculations of the *NNN* system by Rupp and Tjon[2], the electromagnetic form factors were calculated by analogy with three-dimensional descriptions where there is no subtraction term; as a result, these calculations contain overcounting.

Gauging the AGS equations

Although Eqs. (3) and (4) solve the problem of gauging three nucleons, they do require explicit knowledge of the two-body potential v_i and the gauged two-body potential v^μ_i. As it is often preferable to have the input in terms of the two-body t-matrix t_i and the gauged two-body t-matrix t^μ_i, we provide an alternative solution to the gauging problem. Our starting point here shall be the AGS equations describing the scattering of three identical nucleons. The essential point is that the input to these equations consists of two-body t-matrices. For identical particles there is a variety of ways to define the AGS amplitude, all giving the same three-body Green function G. The one we have chosen, which we call Z, satisfies the particularly simple AGS equation for identical particles

$$\tilde{Z} = G_0 + D_{03}t_3\mathcal{P}\tilde{Z} \tag{5}$$

where $\tilde{Z} = G_0 Z G_0$ and where $\mathcal{P}$ shifts particle labels cyclically to the left. As $\tilde{Z}$ is related to the three-body Green function G in a simple way, the solution to our problem of specifying G^μ now rests essentially on the construction of the gauged AGS Green function $\tilde{Z}^\mu$. Gauging Eq. (5) in the same way we gauged Eq. (1), we obtain

$$\tilde{Z}^\mu = \tilde{Z}d_3^{-1}d_3^\mu + \tilde{Z}\left(D_{03}^{-1}D_{03}^\mu D_{03}^{-1}d_3^{-1} + d_3^{-1}t_3^\mu\mathcal{P}\right)\tilde{Z}. \tag{6}$$

This equation describes the attachment of photons at all possible places in the multiple-scattering series of three identical particles. The input is in terms of two-body t-matrices. As such, it forms the central result in the gauged three-nucleon problem.

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1. Kvinikhidze AN, Blankleider B, *Coupling photons to hadronic processes*, invited talk at the Joint Japan Australia Workshop, Quarks, Hadrons and Nuclei, November 15-24, 1995 (unpublished); a more detailed account is in preparation.

2. Rupp G, Tjon JA, Phys. Rev. C **37** (1988) 1729; *ibid.* **45** (1992) 2133.